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ACTL3162 Report

task 2

**1.0 Introduction**

A company’s net assets can be modelled by the following equation:

Where:

* is the initial surplus
* is the premium rate
* is the aggregate amount of losses up to time t

The Cramér-Lundberg model assumes that each is independently and identically distributed.

**2.0 The Probability of Ruin**

Ruin Theory involves using mathematical models to assess an insurance company's risk of going bankrupt. Essentially, the 'probability of ruin' denotes the likelihood of a company, starting with an initial capital of , heading towards insolvency and thus being incapable of sustaining itself, represented as. However, for practical purposes, it's often more beneficial to determine this probability over a specific timeframe, represented as .

**3.0 Proportional vs Excess of Loss Reinsurance**

**3.1 Ruin Probabilities within 5 years**

Reinsurance is a strategy employed to distribute a risk, ensuring that should a claim arise, the primary insurer doesn't bear the full brunt. This principle naturally ties into the probability of ruin. By using reinsurance, the insurer isn't solely responsible for settling the entire claim, which in turn should diminish their risk of insolvency. Let's denote the amount paid by the Reinsurer as h(Y). With a provided loading factor ξ of 0.125 and a retention rate α of 0.7, the Cramér-Lundberg model can be reformulated as follows:

This can be conceptually distilled to a Cramér-Lundberg process without reinsurance, characterised by a rate of 𝜋 − 𝜋h and a claim severity of Y𝑖 − h(Y𝑖) which equals Y𝑖. With this understanding, we can employ R to determine the probability of ruin over a span of 5 years using the "ruin" package. The premium rate can be determined as follows:

The revised claim severity is represented as . By leveraging the scaling trait of the gamma distribution, if Y ~ 𝐺𝑎𝑚𝑚𝑎(5,1.5), then ~ 𝐺𝑎𝑚𝑚𝑎(,1.5), which is equivalent to 𝐺𝑎𝑚𝑚𝑎(3.5,0.5). The “ruin\_probability” function for a 5-year risk of ruin yields a result of 0.003622.

For product B, where h(Y) = (Y − 𝑑)+, it's specified that d = 2.57359. As ruin only transpires when the surplus becomes negative, the strategy is to first simulate a value for 𝑁(60). We then simulate an equivalent number of arrival times from a uniform distribution ranging from 0 to the simulated 𝑁(60) value. This is conducted over 10^5 iterations. The estimated probability of ruin is then derived from the ratio of iterations resulting in ruin, giving a value of 0.00364.

**3.2 Finding the range of and d**

The equations needed to solve to find and d are given below:

For product A:

For product B:

Defining them as functions in R and using the “uniroot” function, the range for d is d > 1.349 and the range for is . Refer to Appendix A for more details.

**3.3 Upper Bounds for Probability of Ruin**

In reinsurance, the adjustment coefficient 'r' should satisfy:

To minimise the ruin probability bound we aim to maximise 'R'.

Refer to Appendix B for the numerical calculations to find the retained proportion and d which maximise the adjustment coefficient. The values obtained from R were and respectively.

The upper bound for the probability of ruin with this choice of is 0.04114744 and the upper bound for the probability of ruin with limit d is 0.04115580 (Refer to Appendix C for results).

**4.0 Trade-off Between Stability and Profitability**

Insurance companies constantly face the challenge of balancing stability and profitability. Proportional and Excess of Loss can enhance an insurer’s stability.

**Proportional Reinsurance (A)**

**Stability:** Through sharing a predetermined percentage of premiums and claims with reinsurers, insurance companies gain a safety net. This partnership allows them to consistently achieve desired financial results and fortify their capital reserves against significant claims (Meyers, 2002).

**Profitability Trade-off:** While the safety net minimises their risk, it also entails sharing lucrative profits with the reinsurer. This means that their financial outcomes, while more stable, might not achieve peak profitability (Schmid, 2007).

**Excess of Loss (EoL) Reinsurance (B)**

**Stability:** EoL reinsurance operates with a defined claim limit. Claims exceeding this limit are the responsibility of the reinsurer, providing insurance companies with a shield against catastrophic losses that could threaten their solvency (Cummins & Harrington, 2003).

**Profitability Trade-off:** Insurance companies benefit by retaining all profits up to the predetermined limit. However, the premiums they pay for this specialized protection could dent their profitability (Wüthrich & Merz, 2008).

**In conclusion:** The role of reinsurance is pivotal in enhancing an insurer's defence against major claims. This added resilience, however, can come at the price of sharing profits or incurring premium costs. Consequently, insurers must make informed decisions, weighing their appetite for risk against market dynamics.

**5.0 Key Scenarios Where Reinsurance is Vital**

**Catastrophic Events:** Events such as tsunamis, earthquakes, or wildfires can generate an enormous volume of claims in a short period. Without reinsurance, such events could potentially bankrupt an insurer. Reinsurance serves as a bulwark, ensuring claims can be met without exhausting reserves (Schmid, 2007).

**Market Expansion:** For insurers eyeing growth in new regions or sectors, the landscape can be fraught with unknown risks. Here, reinsurance steps in as a safety mechanism, protecting them from these uncertainties and also providing seasoned insights to navigate these new waters (Wüthrich & Merz, 2008).

**Financial Stability:** The insurance sector experiences its highs and lows. Reinsurance acts as a moderator, ironing out extreme fluctuations and ensuring steady financial performance. This not only instils confidence in stakeholders but also aids in long-term strategic planning (Schmid, 2007).

**References**

Cummins, J. D., & Harrington, S. E. (2003). Risk and return in the insurance industry. The Journal of Risk and Insurance, 70(4), 665-692.

Meyers, G. (2002). Insurance and reinsurance: Principles and practice. Prentice Hall.

Schmid, J. (2007). Reinsurance for profitability and growth: A new approach to pricing and underwriting. John Wiley & Sons.

Wüthrich, M. V., & Merz, M. (2008). Reinsurance pricing and risk management. John Wiley & Sons.

**Appendices**

**Appendix A**

A close-up of numbers

Description automatically generatedA close-up of a number

Description automatically generated

**Appendix B**

For product A:

For product B:

For A this becomes:

For B this becomes:

**Appendix C**

|  |  |  |
| --- | --- | --- |
| **Option** | **Simulated Ruin Probability** | **Upper Bound** |
| **Proportional** | 0.003534 | 0.04114744 |
| **EOL** | 0.003622 | 0.04115580 |